On the Movement of a Coloured Index along a Capillary Tube, and its Application to the Measurement of the Circulation of Water in a Closed Circuit

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XV. *On the Movement of a Coloured Index along a Capillary Tube, and its Application to the Measurement of the Circulation of Water in a Closed Circuit.* By ALBERT GRIFFITHS,

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§1. *Introduction.*

If a capillary tube, originally full of water, and along which a slow flow is taking place, be fed with a weak fluorescein solution, the point to which the colour reaches can be read, at the speeds studied in this Paper, with tolerable consistency when the illumination is constant ; and as there is a paraboidal flow under certain conditions along a capillary tube the apex of the paraboloid moving at twice the mean speed of the water, a hasty consideration of the subject might lead to the erroneous conclusion that the extremity of the coloured index moves at twice the mean speed, or even more, the excess being due to diffusion.

At the speeds studied in this Paper, it is found that the extremity of the coloured index moves at a speed distinctly less than twice the average speed of the water ; in fact, its movement is a rough measure of the average speed.

In this Paper the full mathematical treatment is not attempted ; but an elementary consideration, although not complete, will be of advantage in dealing with the experiments. It can easily be shown that if the intensity of the colour were constant over a cross-section of the tube the colour would diffuse along the tube exactly as if the water travelled in a solid column. The intensity of the colour can not be absolutely constant over a cross-section except in the theoretical case of a capillary tube of infinitely small bore ; but the experiments of §4 show that when the rate of flow is small the error involved in the assumption is not great. By stopping the supply of fluorescein solution and replacing it by water, an approximately symmetrical column of colour of slowly increasing length can be obtained, and when the slowly moving column is at a relatively long distance from the ends of the fine capillary, it is obvious that the movement of the centre of the column must measure the mean speed of the water.

But even when it is not correct to assume that the intensity of colour is absolutely constant over a cross-section, an approximately symmetrical column, the centre of which indicates the mean speed, can be obtained. This is proved in the next section.

§ 2. *Experiments with a Tube of about 2 mm. Bore.*

A convenient apparatus for the experimental study of the subject is shown in Fig. 1. T is a calibrated tube containing the coloured index, R is a series of fine tubes acting as a resistance, H is a tube containing water, the height of which regulates the rate of flow through the tube T, I is indiarubber

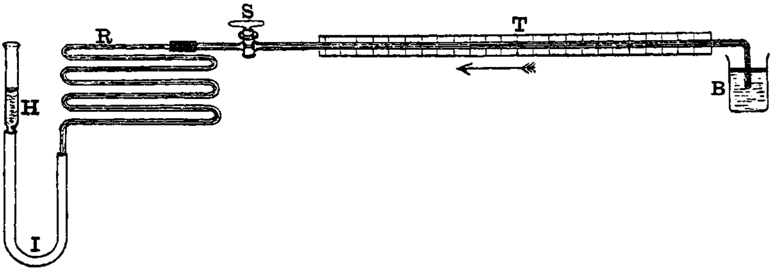

Fig. 1.

tubing, S is a stop-cock, and B is a beaker containing water on which floats a layer of oil to minimise evaporation.

The following table of observations and calculations explains itself :—

Average readings of ends of index, 1 p.m., Nov. 19th	134·32, 144·31
Corrected readings	134·44, 144·22
Mean of corrected readings	139·33 cm.
Weight of beaker, water and oil, 1 p.m., Nov. 19th	53·6800 gms.
Average readings of ends of index, 11 a.m., Nov. 24th ...	52·25, 70·02
Corrected readings	52·38, 69·927
Mean of corrected readings	61·15 cm.
Weight of beaker, oil and water, 11 a.m., Nov. 24th	50·8232 gms.
Correction due to evaporation, obtained with aid of auxiliary beaker	0·0028 gm.
Corrected weight of beaker	50·8260 gms.
Corrected loss of weight of beaker	2·854 gms.
Product of movement (78·18 cm.) of index into mass (0·4964 ÷ 13·56) per linear centimetre	2·859 gms.
Percentage difference....	0·18
A second experiment gave :—	
Corrected loss of weight of beaker	3·8028
Product of movement (104·056 cm.) of index into mass (0·4964 ÷ 13·56) per linear centimetre	3·8099
Percentage difference....	0·19

§ 3. *Details of Index, Methods of Observation and Tests of Consistency.*

The fluorescein solution employed in the experiments of this Paper was made by adding one quarter of a gram of fluorescein and two drops of an aqueous solution of ammonia of strength 0·88 to a litre of distilled water. The tube containing the index is placed on a black scale with white graduation marks.

When readings were taken directly with the eye the source of illumination was an arc lamp taking 12 amperes, placed in a lantern with only the condenser in position. A sheet of ground glass covered the condenser to diffuse the light and to give an even illumination of the index ; and a sheet of blue glass placed in front of the ground glass absorbed a quantity of light without any appreciable diminution of the fluorescence of the index.

The consistency of the readings was tested in the case of a calibrated tube of about 0·25 mm. diameter, by introducing two indexes in series, the linear speed being of the order of 1·5 cm. per hour, or 36 cm. per day.

	Movement of front index. Centimetres.	Movement of back index. Centimetres.	Ratio of front to back.
1	55·39	55·00	1·0071
2	51·63	51·83	0·9960
3	58·27	58·22	1·0009
4	68·18	68·19	0·9999
5	68·25	68·33	0·9988
6	51·14	51·34	0·9961
7	55·94	56·21	0·9952
8	44·95	44·97	0·9996
9	52·48	52·08	1·0077
10	54·55	54·65	0·9981
11	57·47	57·39	1·0014
12	59·47	59·57	0·9983
		Average	0·999925

The probable error of a single value of the ratio, as given by the formula $r=0{\cdot}8453\sum v \div \sqrt{n(n-1)}$, where $\sum v$ is the sum of the arithmetical differences between the values of the ratios and 0·999925, is 0·002561—*i.e.*, about $\frac{1}{4}$ per cent.

The probable error of the arithmetical mean of the ratios, as given by the formula $r=0{\cdot}8453\sum v \div n\sqrt{n-1}$ is 0·0007392—*i.e.*, less than $\frac{1}{10}$ per cent.

On the assumption that all the indexes are equally reliable, whatever their ages, it can be proved that the probable error of an individual reading of the mean position of an index is

0·7 mm. The assumption is not correct; nevertheless, the value 0·7 mm. gives a rough idea of the order of accuracy when no special devices are employed to obtain an exact reading. It must be mentioned that as a rule only one observation of each end of an index was taken.

After the preceding 12 experiments were concluded, an attempt was made to determine the consistency of the reading of the middle point of the index by making two parallel white lines on a strip of black paper at a distance apart approximately equal to the length of the index. The strip of paper was adjusted until the two white lines occupied a symmetrical position with respect to the index. Mr. H. G. Bell, who is responsible for nearly all the readings of this Paper, took six readings with the aid of the paper strip. The probable error calculated from the six readings came out to be 0·36 mm.

A photographic method of taking the readings has been developed which possesses some features of interest. The source of light is the same as that employed for the eye-readings, excepting that the blue glass is removed. Panchromatic plates are used in the camera, with a screen of "Rapid Filter Green." The arc is about 2 ft. from the tube, and the exposure corresponds to that of about 6 seconds with a lens aperture of *f* 11. After the negative has been developed and dried, it is cut close and parallel to the image of the capillary tube. The index is then cut into two approximately equal parts, and the two halves are adjusted until one is as nearly as possible a match of the other.

When the two halves are matched the arithmetical mean of any two corresponding readings gives the centre of the index. The three pictures in Fig. 2 elucidate the method. The uppermost shows the positive of the complete index. The left-hand figure shows the two halves matched, the right-hand figure shows an inaccuracy of matching of 2 mm., the mean reading being inaccurate by 1 mm. The figures refer to an index in a tube of about 1 mm. diameter; the index had travelled for two days at the rate of about $1\frac{1}{4}$ cm. per hour.

Mr. Bell matched the two halves six times in succession. His results were: 28·00, 28·01, 27·995, 28·00, 28·00, 28·00; mean 28·0008 cm.

Mr. F. E. Tinkler, who has frequently assisted Mr. H. G. Bell during the last 12 months, likewise took six readings. His results were: 28·00, 28·025, 27·98, 28·05, 27·96, 28·00; mean 28·0025 cm.

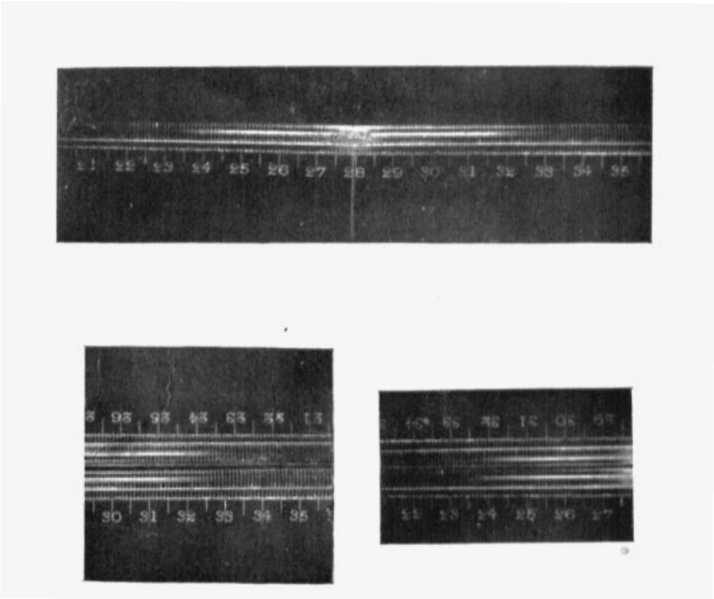

Fig. 2.

Velocity 1·25 cm. per hour; diam. min. ; age 2 days.

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It may be confessed that the negative was a specially good one.

A series of seven pairs of photographs was taken of two indexes in series in a tube of $\frac{1}{4}$ mm. bore. The negatives were not good, and the probable error of an individual reading, as calculated from the series, came out about twice the probable error of a reading taken with the unaided eye; but the author is convinced that the method specified above, which is satisfactory for a tube of 1 mm. bore, is not adapted for the photographing of an index in a tube of $\frac{1}{4}$ mm. bore.

§ 4. *Experiments Relating to the Elementary Theory.*

The tube referred to in section 2 and an auxiliary tube of the same diameter, both being originally full of water, were fed with a solution of fluorescein, the speed of flow being of roughly the same value in each case. After a movement of about 15 cm. had taken place the auxiliary tube was disconnected from the flow apparatus and each end sealed with an india-rubber cap. A fiducial mark was made on the auxiliary tube. Then the auxiliary tube was placed alongside the principal tube and its position adjusted, until the ends of the columns in each tube matched, and a reading was taken of the position of the fiducial mark. The flow was allowed to continue in the principal tube, and after some days the columns were again matched and a reading taken of the fiducial mark. The flow in the principal tube was calculated (*a*) by weighing a beaker and contents, as in § 2, and (*b*) from the product of the movement of the extremity of the coloured column, as measured by the auxiliary tube, into the mass of water per linear centimetre of the tube.

In one experiment the end of the column moved at the comparatively rapid rate of about $1\frac{1}{6}$ cm. per hour through a distance of about 84 cm. It was obvious by inspection that the end of the moving coloured column was more diffused than the end of the column in the auxiliary tube; nevertheless an attempt at matching was made. The flow calculated from the movement of the coloured column as determined by the auxiliary tube was 1·4 per cent. greater than that calculated from the loss of weight of the beaker.

In a second experiment with the same tube the speed was much slower, being about $\frac{1}{5}$ cm. per hour through a range of about 89 cm. The movement as determined by method *b* was

1·04 per cent. greater than that as determined by method *a*. In this experiment there was no characteristic difference between the ends of the two columns.

An experiment was performed with a calibrated tube of $\frac{1}{4}$ mm. bore; here the weighing method of determining the flow is impracticable, and the flow was measured by placing an index in advance of the continuous column. An auxiliary matching tube of $\frac{1}{4}$ mm. diameter was employed to measure the movement of the continuous column. The speed was about $1\frac{1}{4}$ cm. per hour over a range of about 63 cm. The movement of the column as measured by the auxiliary tube was 0·75 per cent. greater than the movement of the index. The reader is reminded that the vertex of the paraboloid moves 100·00 per cent. faster than the index.

Although this section deals particularly with the flow of an indefinitely long column of fluorescein solution, a few words on the determination of the position of a finite index may not be out of place. The author is of the opinion that the best results would be given by the photographic method, if all the details were carefully arranged to suit the tube employed; or else by some equivalent optical device which would enable the two ends to be seen alongside and thus matched directly. At present, however, he would recommend the observer to be content with ordinary eye-readings taken with the assistance of a simple piece of apparatus which is a modification of the strip of black paper mentioned in § 3. It consists of two black metal pieces running along the scale at an adjustable distance apart, each metal piece carrying a vernier, the zero line of which can be brought opposite any desired point near an end of the fluorescein index.

Some experiments, similar to those in § 3, with an index moving at the speed of approximately 4 cm. per hour for about 17 hours in a tube of about 1 mm. diameter, show that with the vernier arrangement the probable error of an individual reading is 0·2 mm. This means that the probable error of a single measurement of the movement is about $\frac{1}{20}$th per cent.

§ 5. *Applications.*

Fig. 3 gives a diagrammatic sketch of an arrangement designed for the introduction of an index into a closed circuit in which there is a circulation of water. T is a capillary tube, R is a small reservoir, A, B, C and D are taps. The ends of the tubes associated with B and C are provided with indiarubber caps. To introduce the index close the taps A and D and run fluorescein solution in at C and out at B. Close the taps B and C, open A and D and allow the flow in the capillary to take place. When the column of fluorescein solution is of the desired length, close the taps A and D, and wash the fluorescein out of the reservoir with water. Now close B and C, and open A and D. *An emphatic warning is given against any trust in glass taps.* If there is a part of the circuit open to the atmosphere, the pressure at the tap A should be determined, and the pressure due to the fluorescein solution, or the pressure due to the water used to wash out the reservoir, should not differ much from the determined value. Otherwise the taps A and D may leak to such an extent as to materially disturb the feeble agency causing the slow circulation. When the circulation is

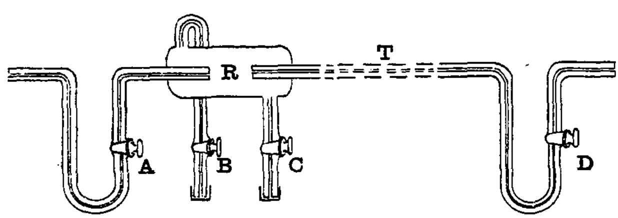

FIG. 3.

taking place, the ends of the tubes associated with B and C are covered with indiarubber caps, and all the taps are immersed in mercury contained in beakers.

A measurement is being made of the coefficient of diffusion of a salt in water by the method of diffusive convection,* and the index (Fig. 2) was obtained with an apparatus designed to apply the method. It is possible that the index may be useful in delicate thermometers and manometers.

All the work recorded in this Paper, except that connected with Fig. 2, was done in a room on the third floor of Birkbeck College. The room has two doors and three windows. It is used by research students, and no attempt has been made to keep a constant temperature. The method of causing a flow along a capillary tube described in § 2 is convenient, but it does not produce a steady flow under the conditions just mentioned, and it is probable that the indexes would be more consistent under superior conditions.

* "Proc." Phys. Soc., Vol. XVI., Part IV., pp. 230-243, Jan., 1899, or "Phil. Mag.," pp. 453-465, Nov., 1898.

In conclusion, the author desires to express his thanks to Mr. H. G. Bell, to whose skill and patience must be largely attributed the measure of success attained in the experiments described in this Paper.

ABSTRACT.

If a fine capillary tube, originally full of water, is fed with a weak fluorescein solution at a rate so rapid that the diffusion of the fluorescein may be neglected, the vertex of the coloured paraboloid moves at double the mean speed of the liquid. But at slow speeds radial diffusion plays an important part, and the colouring matter travels along the tube approximately as if the liquid moved in a solid column. If, after a short length of fluorescein solution has been introduced into the tube, the supply of solution is replaced by water, a symmetrical coloured column of slowly increasing length is obtained, the centre of which indicates the mean speed of the liquid. The illumination is the light from an arc-lamp which has been transmitted through blue glass, and the capillary tube is placed on a black scale with white graduation marks. It is comparatively easy to determine the mean speed of the liquid to an accuracy of $\frac{1}{5}$th per cent.

DISCUSSION.

Dr. A. RUSSELL gave some illustrations of the extreme slowness of diffusion and remarked that he was sorry the author had not given more mathematics in his Paper.

The PRESIDENT remarked that very few realise the extreme slowness of diffusion. He had once worked out a case of a vertical column of a solution under the action of gravity, and found that the time required to pass from a state of uniform density to the final state of equilibrium was well over a million years.